

Back-reaction of the Hawking radiation flux on a gravitationally collapsing star II: Fireworks instead of firewalls

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A star collapsing gravitationally into a black hole emits a flux of radiation, known as Hawking radiation. When the initial state of a quantum field on the background of the star, is placed in the Unruh vacuum in the far past, then Hawking radiation corresponds to a flux of positive energy radiation travelling outwards to future infinity. The evaporation of the collapsing star can be equivalently described as a negative energy flux of radiation travelling radially inwards towards the center of the star. Here, we are interested in the evolution of the star during its collapse. Thus we include the backreaction of the negative energy Hawking flux in the interior geometry of the collapsing star and solve the full 4-dimensional Einstein and hydrodynamical equations numerically. We find that Hawking radiation emitted just before the star passes through its Schwarzschild radius slows down the collapse of the star and substantially reduces its mass thus the star bounces before reaching the horizon. The area radius starts increasing after the bounce. Beyond this point our program breaks down due to shell crossing. We find that the star stops collapsing at a finite radius larger than its horizon, turns around and its core explodes. This study provides a more realistic investigation of the backreaction of Hawking radiation on the collapsing star, that was first presented in [1].

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I. INTRODUCTION

The backreaction of Hawking radiation on the evolution of the collapsing star is the most important problem in the quantum physics of black holes. This problem provides an arena for the interplay of quantum and gravitational effects on black holes and their respective implications for the singularity theorem. A key feature of Hawking radiation, which was well established in seminal works by [3, 4, 9, 11, 17–20], is that the radiation is produced during the collapse stage of the star prior to black hole formation. The very last photon making it to future infinity and thus contributing to Hawking radiation, is produced just before an horizon forms. However its effect on the collapse evolution of the star was considered for the first time only recently [1]. As was shown in [1] once the backreaction of Hawking radiation is included in the interior dynamics of the star, then the collapse stops and the star bounces. Solving analytically for the combined system of a collapsing star with the Hawking radiation included, is quite a challenge. The system studied in [1] was idealized in order to obtain an approximate analytical solution: there the star was taken to be homogeneous; the star's fluid considered was dust; the star was placed in a thermal bath of Hawking radiation which arises from the time-symmetric Hartle-Hawking initial conditions on the quantum field in the far past. Within these approximations, the main finding of [1] was that a singularity and an horizon do not form after the star's collapse because the star reverses its collapse and bounces at a finite radius due to the balancing pressure of the negative en-

ergy Hawking radiation in its interior. Yet, the evolution of the star could not be followed beyond the bounce with the approximate analytic methods of [1].

Given the fundamental importance of this problem and the intriguing results of [1], we here aim to study the backreaction of Hawking radiation on the collapsing star by considering a more realistic setting, namely: we allow the star to be inhomogeneous and consider an Hawking radiation flux of negative energy which propagates in the interior of the star, with its counterpart of positive energy flux travelling outwards to infinity. Hawking radiation flux arises when the initial conditions imposed on the quantum field on the background of the star, are chosen to be in the Unruh vacuum state in the far past [9, 18]. In contrast to the Hartle-Hawking initial state which leads to an idealized time symmetric thermal bath of radiation present before and after the collapse, the choice of the Unruh vacuum describes a flux of thermal radiation which is zero before the collapse and switches on after the collapse. We solve numerically the full 4 dimensional set of Einstein and of total energy conservation equations leading to a complete set of hydrodynamic equations for this model. Numerical solutions allows us to follow the evolution of the collapsing star beyond its bounce.

The paper is organized as follows: Section II describes the metric in the star's interior, the stress energy tensor for the star and for the Hawking radiation flux, which comprise the model we wish to study. We then set up the system of the evolution equations that need to be solve for the combined system. In Sec. III we describe the numerical implementation and the two codes written,

and provide a consistency test for the codes by applying them to the well known Oppenheimer Snyder model. In Sect. IV we provide the results of the numerical solutions for the evolution of the interior geometry of the star and conclude in Sect. V.

II. MODEL

Let us start with a spherically symmetric and inhomogeneous dust star, described by the following metric

$$ds^2 = -e^{2\Phi(r,t)} dt^2 + e^\lambda dr^2 + R^2 d\Omega^2 \quad (1)$$

where $R(r,t)$ is the areal radius and $d\Omega^2 = d\theta^2 + \sin^2(\theta) d\phi^2$. This form of the metric is convenient for describing a radiating star. The set of Einstein and energy conservation equations, were originally derived in Misner [12]. We will use the set of equations [12] and [10], jointly known as the hydrodynamic equations, by modifying them to reflect our particular system. At the onset of collapse the star gradually starts producing a flux of Hawking radiation. The positive energy flux travels outwards to future infinity and an equivalent but negative energy flux travels radially inwards in the interior of the star towards the center. Most of the outgoing null radiation is produced on the last stages of collapse [9, 11, 19, 20]. We wish to include the backreaction of the negative energy Hawking radiation flux on the interior of the star and solve for its evolution.

The stress energy tensor of the Hawking radiation flux is given by

$$\tau^{ab} = q_H k^a k^b \quad (2)$$

where k^a is an outgoing null vector $k^a k_a = 0$ and q_H is the Hawking radiation energy density related to the luminosity of radiation by $L = 4\pi R^2 q_H$.

The fluid of the inhomogeneous dust star with a 4-velocity u^a , normalized such that $u^a u_a = -1$, has a stress energy tensor

$$T^{ab} = \epsilon u^a u^b, \quad (3)$$

where the energy density ϵ is expressed in terms of a specific internal energy density per baryon e and the number density of baryons n by $\epsilon = n(1 + e) = nh$ with h the enthalpy. We normalize k_a by the condition $k_a u^a = 1$, so that q_H denotes the magnitude of Hawking energy flux density of the inward moving radiation in the rest frame of the fluid. Therefore the total stress energy tensor of the combined system of the star's fluid (T_{ab}) and of the radiation (τ_{ab}) is

$$T_{total}^{ab} = T^{ab} + \tau^{ab} \quad (4)$$

The equations that describe the dynamics of the interior of the star with the backreaction of the Hawking radiation flux included are: the Einstein equations,

$$G^{ab} = T_{total}^{ab}, \quad (5)$$

the total energy conservation equations,

$$\nabla_b T_{total}^{ab} = 0, \quad \nabla_b T^{ab} = -\nabla_b \tau^{ab}, \quad (6)$$

and the baryon number conservation

$$\nabla_a (n u^a) = 0. \quad (7)$$

The form of the renormalized stress energy tensor for the Hawking radiation flux in the Unruh vacua, at future infinity and near the surface of the star were calculated in Candelas [18]. Based on energy conservation, the net inward negative energy flux in the star's interior is equal and opposite to its value at infinity, shown in [3, 18, 20, 23] who also showed that the Unruh vacuum best approximates the state that follows gravitational collapse since it is such that it is nearly empty in the far past before the star collapses, but then produces a flux of radiation once the collapse starts. On these basis the quantum mechanical stress energy tensor of the Hawking radiation flux in the interior is as follows

$$\tau_a{}^b = \frac{L}{R^2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (8)$$

The total energy conservation is best written as a set of two equations which contain the radiative heat transfer from the fluid to the Hawking flux, its t-component and the r-component that are obtained by contracting T^{ab} in 6 with u^a and the unit vector n^a orthogonal to the 3-hypersurface Σ_t at $t = \text{constant}$ as follows

$$Q^a = \nabla_b \tau^{ab} = (e^{-\Phi} n C, e^{-\lambda/2} n C_r, 0, 0) \quad (9)$$

From Eqn. 9 and the Hawking flux stress energy tensor in the interior Eqn. 8, it is immediately clear that $C_r = -C$. The latter simply reflect the fact that the star is receiving negative energy transfer travelling inwards in its interior.

We can now write explicitly the full set of hydrodynamic equations, given above. First let us define the proper derivatives D_r, D_t by the following relation in terms of the metric

$$D_r = e^{-\lambda/2} \partial_r, \quad D_t = e^{-\Phi} \partial_t. \quad (10)$$

The evolutionary equations for baryon number density $n(r,t)$ from 7, specific internal energy e from 6, the energy of null-radiation with luminosity L given from the t-component of 9 and 6, the proper velocity of the co-moving fluid U , areal radius of comoving fluid elements $R(r,t)$, and the 'acceleration' equation obtained from the

r-component of 9 and the Einstein equations 5

$$\frac{1}{n}D_t n + \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 U) = 4\pi R q_H \alpha f^{-1/2}, \quad (11a)$$

$$D_t e = -C - p D_t \left(\frac{1}{n} \right), \quad (11b)$$

$$D_t L = 4\pi R^2 n C - e^{-2\Phi} D_r (L e^{2\Phi}) - 2L \left(\frac{\partial U}{\partial R} - \frac{L\alpha}{R f^{1/2}} \right), \quad (11c)$$

$$D_t R = U, D_r R = f, \quad (11d)$$

$$D_t U = f \frac{\partial \Phi}{\partial R} - \frac{m}{R^2} + \frac{L}{R}. \quad (11e)$$

Two auxiliary quantities are given by radial ODEs,

$$\frac{\partial m}{\partial R} = 4\pi R^2 \left(\varepsilon - q_H \left(1 + \alpha U f^{-1/2} \right) \right), \quad (11f)$$

$$(\varepsilon + p) \frac{\partial \Phi}{\partial R} = -\frac{\partial p}{\partial R} + n C f^{-1/2}, \quad (11g)$$

with boundary conditions

$$m = 0, L = 0, \quad \text{at } r = 0, \text{ for all } t, \quad (12)$$

$$\Phi = 0 \quad \text{at } r = r_{\text{star}}, \text{ for all } t. \quad (13)$$

Finally, ε , f , α and R' are short-cuts for

$$f = 1 + U^2 - \frac{2m}{R}, \quad (14)$$

$$\varepsilon = n(1 + e), \quad (15)$$

$$\alpha = \frac{R'}{|R'|} = \text{sign} R', \quad (16)$$

$$R' = \frac{\partial R}{\partial r}. \quad (17)$$

A. Choice of energy conversion rate C

The remaining quantities pressure p and the heating transfer rate C are free data, that must be specified through their functional dependence on the evolved variables. For the present work, we assume pressure-less dust, i.e. $p = 0$ throughout. In the future we will extend the analysis to the case $p = w\varepsilon$ with $w \neq 0$ the equation of state for the fluid.

Having to specify C is somewhat inconvenient, as our goal is a solution of Eqs. (11a)–(11e) for the scenario where the luminosity at infinity is equal to the expected Hawking radiation:

$$L_\infty = L_{H,S}(t)(U_S + \sqrt{f_S})^2 = L_H. \quad (18)$$

where $L_{H,S}(t)$ is the luminosity of the outgoing null radiation at the moving surface of the star and L_H the Hawking flux at future infinity. Unfortunately, Eqs. (11a)–(11e) do not allow to specify L freely. Rather, L follows from the choice of the energy-conversion rate C . Therefore, our task is to choose a suitable function $C(r, t)$.

We being the motivation of our choice $C(r, t)$ by noting that for $p = 0$, Eq. (11b) simplifies to $\partial_t e = -C e^\Phi$. Therefore, we can achieve a spatially constant e (i.e. spatially uniform mass evaporation) with the choice

$$C(r, t) = e^{-\Phi(r, t)} \bar{C}(t), \quad (19)$$

where $\bar{C}(t)$ is a not yet determined function of time.

To determine $\bar{C}(t)$, we consider Eq. (11c), which relates the luminosity L to the energy-conversion rate C . If we disregard all gradient terms and any other term independent of C , then Eq. (11c) becomes $\partial_t L = 4\pi R^2 e^\Phi n C$. This represents the radiation creation rate in a spherical shell at radius R . Integrating over radius with volume element $e^{\lambda/2} dr$, we find a total radiation creation rate of

$$C_{\text{total}} = \int_0^{r_s} dr 4\pi R^2 e^\Phi n C e^{\lambda/2} \quad (20)$$

$$= \bar{C}(t) \int_0^{r_s} dr 4\pi R^2 n \frac{\alpha R'}{\sqrt{f}}. \quad (21)$$

If we disregard radiation propagation effects, i.e. assume that the star is unchanged during a light-crossing time, then we expect that $L_S \approx C_{\text{total}}$. Combining this with Eq. (18), we arrive at

$$L_H = \bar{C}(t) \left(U_S + \sqrt{f_S} \right)^2 \int_0^{r_s} dr 4\pi R^2 n \frac{\alpha R'}{\sqrt{f}}, \quad (22)$$

from which it follows that

$$\bar{C}(t) = \frac{L_H}{(U_S + \sqrt{f_S})^2} \left(\int_0^{r_s} dr 4\pi R^2 n \frac{\alpha R'}{\sqrt{f}} \right)^{-1}. \quad (23)$$

III. NUMERICAL IMPLEMENTATION

The numerical implementation will use coordinates t and r . t represents the proper time of the fluid-element on the surface of the star (this identification is enforced through the boundary condition (13)). r is the radial coordinate comoving with the fluid. Using r as radial coordinate in the numerical implementation ensures that the star always covers the same range $r \in [0, r_{\text{star}}]$. We rewrite Eqs. (11) with partial derivatives according to Eq. (10).

A. Discretization

We implement Eqs. (11) in a finite-difference code, using the Crank-Nicholson method. A uniform grid in comoving radius r is employed. Spatial derivatives are discretized with second order accuracy using centered finite-difference stencils in the interior and one-sided stencils at the boundary. Denoting the vector of evolved variables with $\mathbf{u} = \{n, e, L, R, U\}$, then discretized equations have

the form

$$\frac{\mathbf{u}(r_i, t + \Delta t) - \mathbf{u}(r_i, t)}{\Delta t} = \frac{1}{2} (\mathbf{R}[\mathbf{u}](r_i, t) + \mathbf{R}[\mathbf{u}](r_i, t + \Delta t)). \quad (24)$$

Here, $\mathbf{R}$ represents the right-hand-sides of Eq. (11). $\mathbf{R}$ involves second order finite-difference stencils (centered in the interior, one-sided at the boundary). When evaluating $\mathbf{R}$ at the origin, $r = R = 0$, the rule of L'Hospital is used to evaluate terms that would lead to a division by zero, e.g $L/R \rightarrow 0$, $U/R \rightarrow \partial_r U / \partial_r R$.

We solve Eq. (24) separately for each of the variables n, e, L, R, U , fixing all other variables to guesses of their values at the new time. Once all variables have been solved for, the auxiliary variables f, m , and Φ are updated. Now the updated variables are copied into the guesses, and if necessary the procedure is repeated. We iterate until the maximal norm of the correction is less than 10^{-10} . This takes typically 3-6 iterations.

When integrating Eq. (11f), care must be taken to correctly represent the $m \propto r^3$ behavior close to the origin. A second order finite-difference method cannot resolve this cubic behavior of $m(r)$, and would therefore lead to a loss of accuracy. Instead, we define the auxiliary quantity

$$\eta \equiv \frac{3m}{4\pi R^3}, \quad (25)$$

which is approximately constant close to the origin. We rewrite Eq. (11f) as

$$\frac{\partial \eta}{\partial R} = \frac{3}{R} \left[\varepsilon - q \left(1 + \alpha U f^{-1/2} \right) - \eta \right], \quad (26)$$

and discretize the partial derivative directly in terms of the areal radius R . After solving for η , we recover $m = \frac{4}{3}\pi R^3 \eta$.

Figure 1 demonstrates the expected second order convergence of this finite-difference code.

B. Diagnostics

Besides monitoring the evolved variables and the auxiliary variables, we utilize two additional diagnostics.

First, we compute the expansion $\theta_{(k)}$ of $r = \text{const}$ surfaces along the outgoing null-normal $k^\mu = D_t + D_r = e^{-\Phi} \partial_t + e^{-\lambda/2} \partial_r$. This quantity is computed as

$$\theta_{(k)} = \frac{2}{R} \left(\alpha \sqrt{f} + U \right). \quad (27)$$

When $\theta_{(k)} = 0$, an apparent horizon has formed. Specifically, we will plot $\Theta_{(k)} R/2$, a quantity which is unity in flat space.

Second, we track during the evolution outgoing null geodesics, to gain a deepened understanding of the causal structure of the space-time. The null-geodesics are represented by the level-sets of an auxiliary variable $\Lambda(t, r)$,

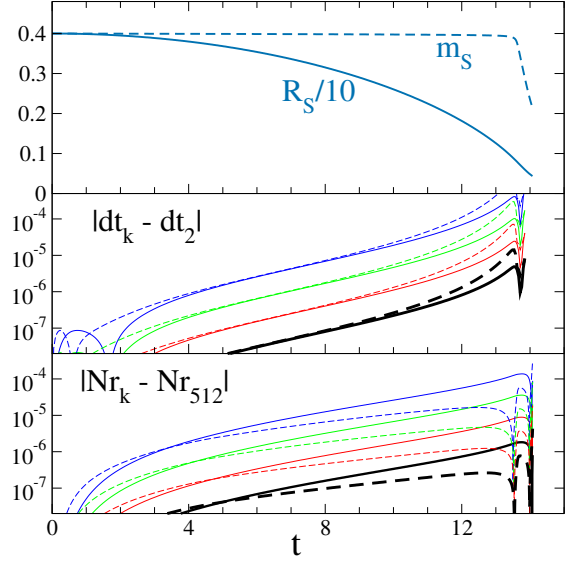


FIG. 1: Convergence test for the numerical code. The top panel shows the evolution of areal radius R_S and total mass m_S for the run discussed in detail in Sec. IV. This evolution was performed at five different time-steps, $dt = 2, 4, 8, 16, 32$ (in units of $\tau_{\text{collapse}}/10000$), and the middle panel shows the difference between runs at the larger for time-steps and the smallest time-step. The evolution was also performed at five different radial resolutions ($Nr = 32, 64, 128, 256, 512$), and the lower panel shows the difference between the finest resolution and the coarser ones. The thick black lines in the lower two panels are representative of the errors in the runs discussed in this paper.

i.e. the lines $\Lambda(r, t) = \text{const}$ are null. To achieve this, Λ must satisfy

$$0 = k(\Lambda) = e^{-\phi} \frac{\partial \Lambda}{\partial t} + e^{-\lambda/2} \frac{\partial \Lambda}{\partial r}. \quad (28)$$

Solving for $\partial \Lambda / \partial t$ results in an evolution equation for Λ ,

$$\frac{\partial \Lambda}{\partial t} = -e^{\phi} \frac{\alpha \sqrt{f}}{R'} \frac{\partial \Lambda}{\partial r}. \quad (29)$$

This is an advection equation with positive advection speed, i.e. the characteristics of Λ (the null rays) always move toward larger values of r . This is an expected result for a comoving coordinate r . Because Λ is always advected outward, we must supply a boundary condition at $r = 0$. This boundary condition sets the value of Λ for the null ray starting at $(t, 0)$, and we choose

$$\Lambda(t, 0) = t. \quad (30)$$

We also need initial conditions at $t = 0$, where we set

$$\Lambda(0, r) = -r. \quad (31)$$

These initial- and boundary conditions ensure that $\Lambda \leq 0$ represents null rays that intersect the $t = 0$ surface, whereas $\Lambda \geq 0$ represent null rays that emanate from

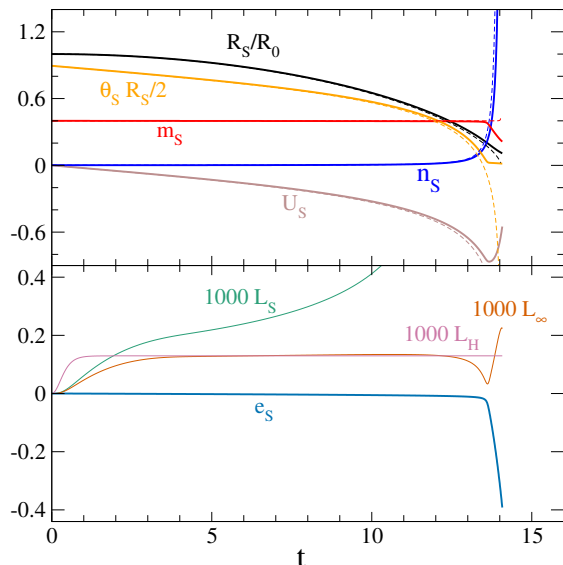


FIG. 2: Evolution of pressure-free collapse with Hawking radiation ($R_0 = 4$, $M_0 = 0.4$). The thick lines indicate the evolution of quantities evaluated at the stellar surface: areal radius R_S , radial velocity U_S , total mass m_S , number density n_S , expansion θ_S . The lower panel shows the internal energy e_S , luminosity at the stellar surface L_S , luminosity at infinity L_∞ , and our target luminosity L_H . For easy of plotting, the luminosities are scaled by a factor of 1000. The thin dashed lines in the upper panel represent the respective quantities in a pure Oppenheimer-Snyder collapse of the same initial conditions.

the origin at coordinate time $t = \Lambda$. The boundary conditions (30) and (31) are consistent with the evolution equation (29), because at the origin $\phi(0,0) = 0$, $f(0,0) = \alpha(0,0) = R'(0,0) = 1$. Equation (29) is added as an extra evolution equation to Eqs. (11).

IV. RESULTS

We will investigate stars with different initial radius R_0 and with different initial masses M_0 . Figure 2 shows a rather typical outcome, a star with initial compactness $R_0/M_0 = 10$. The evolution proceeds first through a phase which is almost identical to standard Oppenheimer-Snyder collapse: The star collapses with increasing inward velocity. The luminosity is very small in this early phase, resulting in a negligible change in internal energy e and mass m_s . The upper panel of Fig. 2 also shows the various quantities for a standard Oppenheimer-Snyder collapse (dashed lines), which essentially overlap the one for the Hawking-radiation collapse case.

The similarities between Oppenheimer-Snyder collapse and the Hawking-radiation collapse end, however, as the star approaches its Schwarzschild-radius, which would happen when $\theta_{(k)}$ crosses zero. Just before the star crosses its Schwarzschild-radius, the luminosity L_S di-

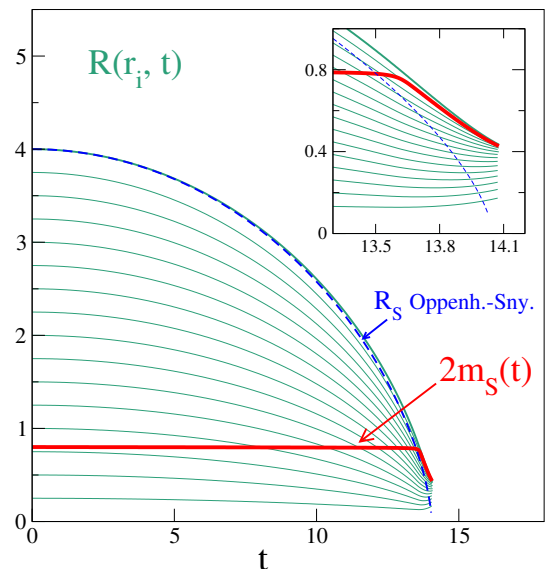


FIG. 3: Evolution of pressure-free collapse with Hawking radiation ($R_0 = 4$, $M_0 = 0.4$). Shown are the area radii at sixteen equally-spaced comoving coordinate radii r_i . The thick red line indicates $2m_S$, which is scaled such that $R_S = 2m_S$ would indicate the formation of a black hole. The thin dashed blue line indicates the areal radius of a Oppenheimer-Snyder collapse.

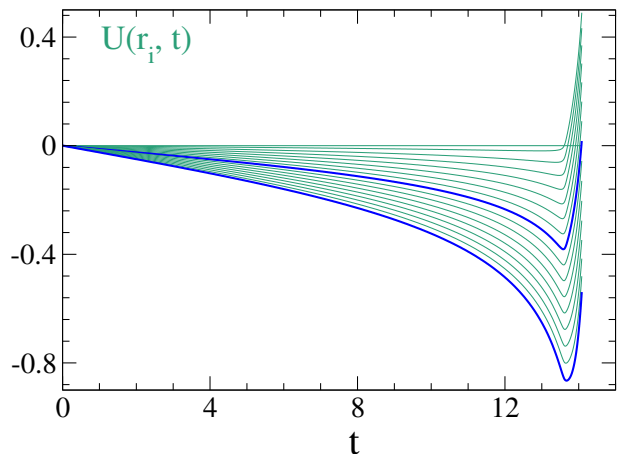


FIG. 4: Radial velocity U for the case $R_0 = 4$, $M_0 = 0.4$. Shown are the area radii at sixteen equally-spaced comoving coordinate radii r_i , with the blue thick lines indicating the stellar surface, $r = r_S$ and the radius $r = r_S/2$.

verges, resulting in substantial reduction of internal energy e and the total mass m_S . The internal energy drops to $e \sim -0.4$, and the total mass drops to about half its initial value.

As can be seen, the velocity of the fluid changes behaviour from negative to positive at the bounce radius, describing a collapsing phase switching to an expanding phase of the fluid. The inner layers of the star pick up a larger positive velocity than the outer layers, resulting in shell crossing. At that point, due to shell crossing which

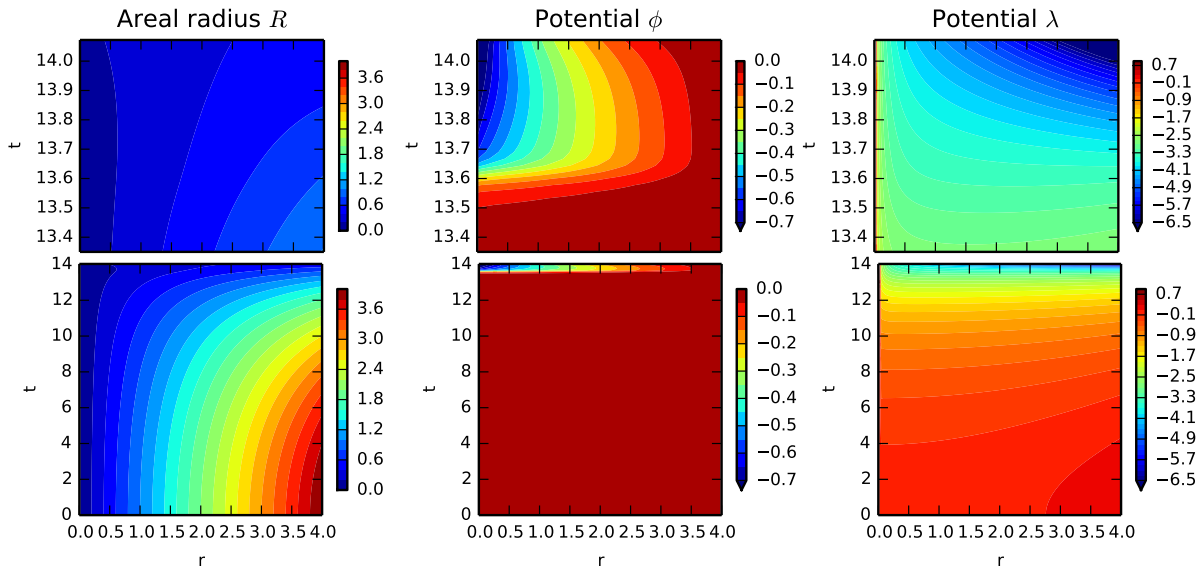


FIG. 5: Contour plots of metric quantities for the simulation with $R_0 = 4, M_0 = 0.4$. The lower panel show the entire time-range on the y-axis, the upper panels zoom into the late-time phase with strong Hawking radiation.

is an artifact of $p = 0$ choice in this model, the density $n(r, t)$ at the surface of the star diverges $\propto (t_{\text{crit}} - t)^{-1/2}$, and the simulation crashes. The divergence in $n(r, t)$ at finite stellar radius and mass indicating shell-crossing of the outer layers of the star, can not be handled by our code. A more sophisticated numerical method, in combination with a fluid-description with pressure would be needed to proceed further and will be done in future work. Nevertheless, we consider the behavior before shell-crossing as generic.

The behavior of this collapse with Hawking radiation is explored further in Fig. 3. Shown are 16 areal radii (at constant comoving coordinates), as well as $2m_S(t)$. The main panel shows clearly the homologous collapse at early times $t \lesssim 13.5$. The inset shows an enlargement of the late-time behavior, showing clearly that the mass m_S is reduced such that the outer surface of the star remains just outside its horizon.

Figure 4 shows radial velocity U at 16 comoving radii. When Hawking radiation becomes dominant during the last ~ 1 of the evolution, the inward motion is significantly slowed down in the outer layers, and even reversed in the inner layers of the star. The radial gradient in the acceleration (with larger outward velocities toward the center of the star) is responsible for the diverging number density in the star. To proceed further will either require a treatment of shocks, or a modification of our model to avoid shocks.

Figure (5), finally presents the metric quantities for this evolution. The areal radius R is very similar to the standard Oppenheimer-Snyder collapse; the small deviations during the Hawking radiation phase are not easily visible. The lapse-potential Φ clearly shows two distinct phases: During the quasi Oppenheimer-Snyder collapse $\Phi \approx 0$. When the Hawking radiation becomes significant,

Φ drops quickly to values around -0.5 . The potential λ shows also two distinct phases: During the Oppenheimer-Snyder phase, λ remains approximately spatially constant, but slowly decreases. During the Hawking radiation phase, λ quickly decreases close to the surface of the star, presumably in accord with the diverging number density n close to the stellar surface.

Figure 6 explores the parameter space of different R_0 and M_0 . Generically, we find the behavior shown in Figs. 2 and 3: The stars first collapse at nearly constant mass until they reach a size close to the horizon (in Fig. 6, this represents a diagonal line at $m_S = 2R_S$). As the stars approach their horizons, luminosity increases and the mass drops significantly, with the star remaining just outside $R_S = 2m_S$. All trajectories approach the same limiting behavior, thus we find the expected generic behavior of Hawking radiation.

We have shown results here for very compact stars as those are the more likely ones to collapse into a black hole. We conducted the analysis for stars with low mass, see the $M = 0.1$ data in Fig. 6. As expected these stars behave differently, and substantially evaporate and explode before they approach their horizon.

V. CONCLUSIONS

Einstein equations tell us that the final destiny of a gravitationally collapsing massive star is a black hole [2]. This system satisfies all the conditions of the Penrose Hawking singularity theorem [25]. However a collapsing star has a spacetime dependent gravitational field which by the theory of quantum fields on curved spacetime should give rise to a flux of particles created [11]. Hawking discovered in the early '70's that this is in-

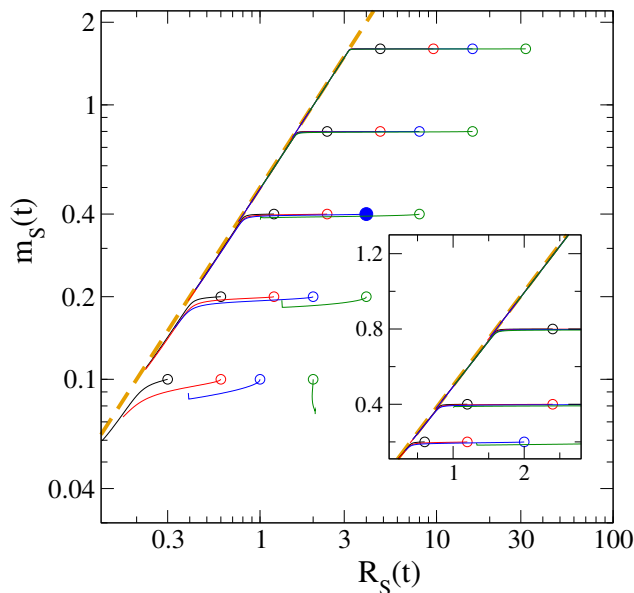


FIG. 6: Collapsing stars with Hawking radiation for several choices of initial mass and initial radius. Shown are trajectories in the $(R_S(t), m_S(t))$ -plane, with the circles indicating the initial conditions of each run. The filled blue circle marks the evolution discussed in detail in Figs. 2–5. The dashed orange line indicates the Schwarzschild horizon, $R_S = 2m_S$. The inset shows an enlargement, to emphasize the universal nature of the near-horizon behavior.

deed the case for stars collapsing to a black hole, they produce Hawking radiation [3]. The conclusions derived from both theories, the existence of black holes from Einstein’s theory of gravity and the existence of Hawking radiation from the theory of quantum fields in curved spacetime, were soon found to be in high friction with one another, (see [26] for an interesting treatment). They led to a series of paradoxes, most notably the information loss [5] and firewalls [7]. Being forced to give up on either unitarity or causality is at the heart of the problem. But the quantum theory is violated if unitarity is broken. Einstein’s theory is violated if causality is broken. Violations of the quantum theory imply Hawking radiation may not exist. Violations of Einstein’s theory of gravity, on which the singularity theorem is based, imply black holes may not exist. Thus black hole physics provides the best arena for understanding the friction between quantum and gravitational physics. In this light, an investigation of the evolution of the star’s interior as it is approaching its singularity with the backreaction of the Hawking radiation produced by quantum effects, is of fundamental importance.

This problem was first investigated with semianalytical methods and a series of approximations, such as a homogeneous star placed in the Hartle Hawking thermal bath in [1].

In this work we extend our investigation to study a more realistic system of a collapsing star with the quantum field in the Unruh vacuum, which gives rise to a

Hawking flux produced only during the collapse. Here we solved the full set of Einstein and energy conservation equations numerically therefore the exact solutions we obtain are robust. We find for a range of initial masses and radii of the collapsing star that the evolution proceeds through two phases: First, a collapsing phase where Hawking radiation is unimportant, and the star follows very closely standard Oppenheimer-Snyder collapse. When the star approaches formation of an horizon, then Hawking radiation sets in. This slows down the collapse (e.g. Fig. 4) while significantly reducing the mass of the star. Both effects (slowdown and mass-loss) balance such that the evaporating star remains very slightly outside its event horizon, cf. Fig. 6.

The key idea that enables this program is the fact that radiation is produced during the collapse stage of the star, prior to black hole formation. Once the star becomes a black hole nothing can escape it, not even light. Since Hawking radiation is produced prior to black hole formation then its backreaction on the star’s evolution can be included in the set of hydrodynamic equations for the coupled system of the quantum field and the star. Thus we solve numerically the set of hydrodynamics equations describing the evolution of collapse for the inhomogeneous dust star absorbing negative energy Hawking flux during its collapse. We discover that the stars explode instead of collapsing to a black hole, as they get close to their last stage of collapse, just when Hawking radiation reaches a maximum, but they never cross what would have been the apparent horizon. Just like Hawking radiation which is universal, we discover that this behaviour of the collapsing stars bouncing and exploding before the horizon and singularity would have formed, is *universal*, i.e independent of their characteristics such as mass and size. Physically the backreaction of ingoing negative energy Hawking radiation reduces the gravitational binding energy in the star with the maximum loss near the last stages of collapse, while taking momentum away from the star. This is the reason for the universal feature of the explosion in the star instead of its collapse to a black hole singularity. Independent of what size and mass the star starts from, most of its radiation will be produced as the star nears its future horizon. At that stage the drop in mass and internal energy is maximum and the star goes through a bounce from collapse to explosion. The negative energy Hawking radiation absorbed by the star, violates the energy condition of the singularity theorem [25] thus it is not surprising that a singularity and an horizon do not form, features traditionally associated with the definition of black holes. Stated simply our findings indicate that singularities and horizon do not exist due to quantum effect and that universally stars blow up on their last stage of collapse.

More specifically, we find that collapsing stars slow down their collapse right outside their horizon, while substantially reducing their mass through Hawking radiation. In the cases studied, the mass of the black hole is reduced by roughly a factor of 2, with the radius of the

star shrinking in proportion, to preserve $R_S/m_S > 2$, cf. Figs. 3 and 6. The velocity of the collapsing matter then reverses toward an *expanding* phase, cf. Fig. 4 with the inner layers expanding faster than the outer layers. Unfortunately the latter behaviour, i.e. the ‘un-folding’ of the star from inside out leads numerically to shell-crossing which our code can not handle. Once shell crossing occurs we can not follow the evolution further to determine whether in the expanding phase, the stellar remnant explode, or simply evaporate away, despite that from the low mass cases and from the velocity results the explosion seems to be the case.

The star never crosses its horizon, so neither unitarity nor causality are violated, thereby solving the longstanding information loss paradox. This investigation shows

that universally collapsing stars bounce into an expanding phase and probably blow up, instead of collapsing to a black hole. Thus ‘fireworks’ should replace ‘firewalls’.

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